What is the Relationship between Tensor Factorizations and Circuits?

Paper

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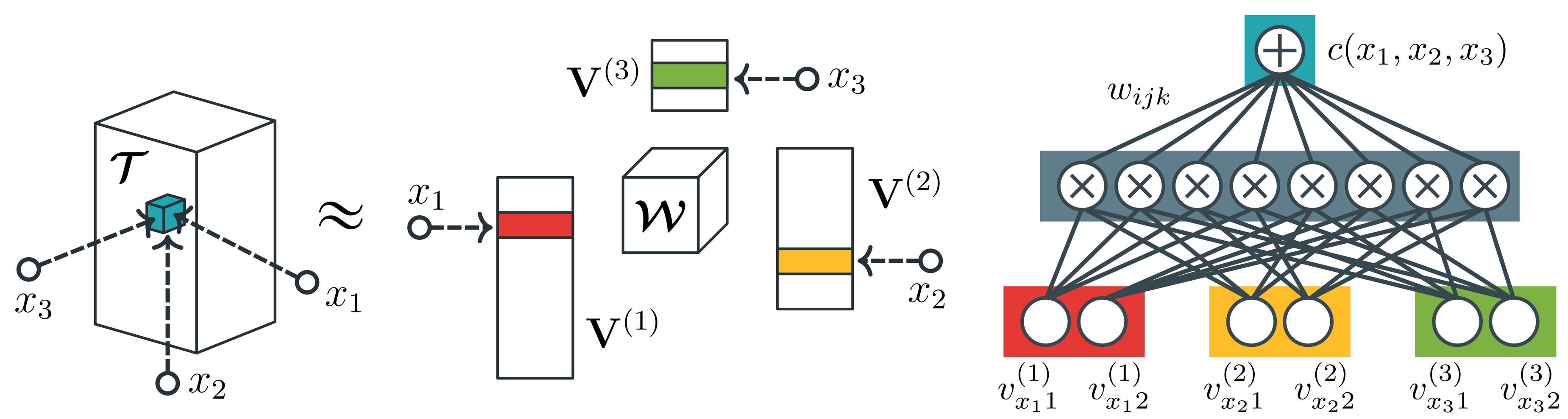
TL;DR:

"We connect tensor factorizations and circuits, present research opportunities for both fields, and provide a pipeline for building and optimizing factorizations and tractable probabilistic models."



$$t_{x_1x_2x_3} \approx \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} \sum_{r_3=1}^{R} w_{r_1r_2r_3}\, v^{(1)}_{x_1r_1}\, v^{(2)}_{x_2r_2}\, v^{(3)}_{x_3r_3}$$

Tensor factorizations such as Tucker are circuits... [1]

$$t_{x_1x_2x_3} \approx c(x_1, x_2, x_3)$$

...and circuits encode factorizations as computational graphs

Input units $c_n(X) = f_n(X;\,\theta_n) \in \mathbb{C}$

Product units $c_n(\mathbf{X}_n) = \prod_{i\in \mathsf{in}(n)} c_i(\mathbf{X}_i)$

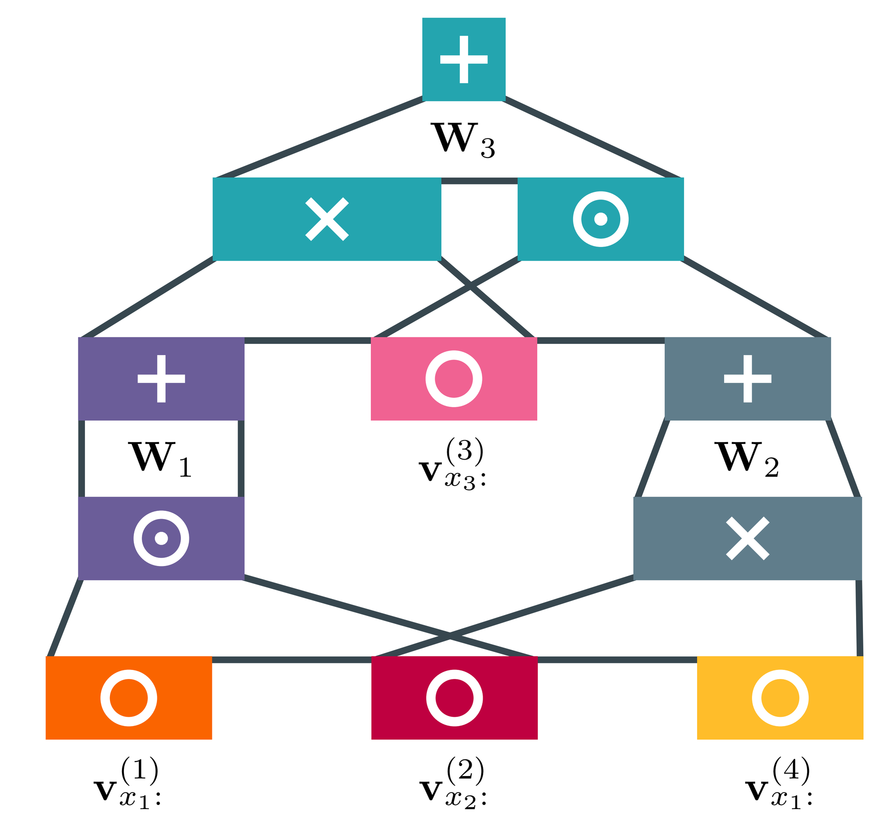
Sum units $c_n(\mathbf{X}_n) = \sum_{i\in \mathsf{in}(n)} w_{n,i}\, c_i(\mathbf{X}_i),\ \ w_{n,i} \in \mathbb{C}$

Probabilistic circuits: $t_{x_1\cdots x_d} = p(x_1,\ldots,x_d) = Z^{-1} c(x_1,\ldots,x_d)$

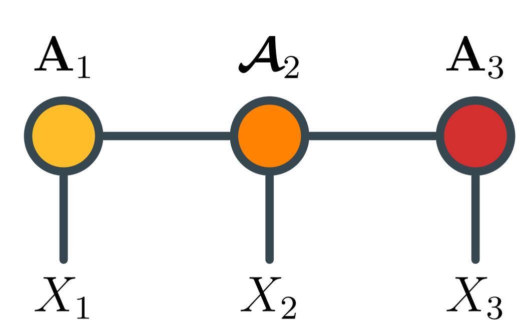
A framework for tractable probabilistic inference

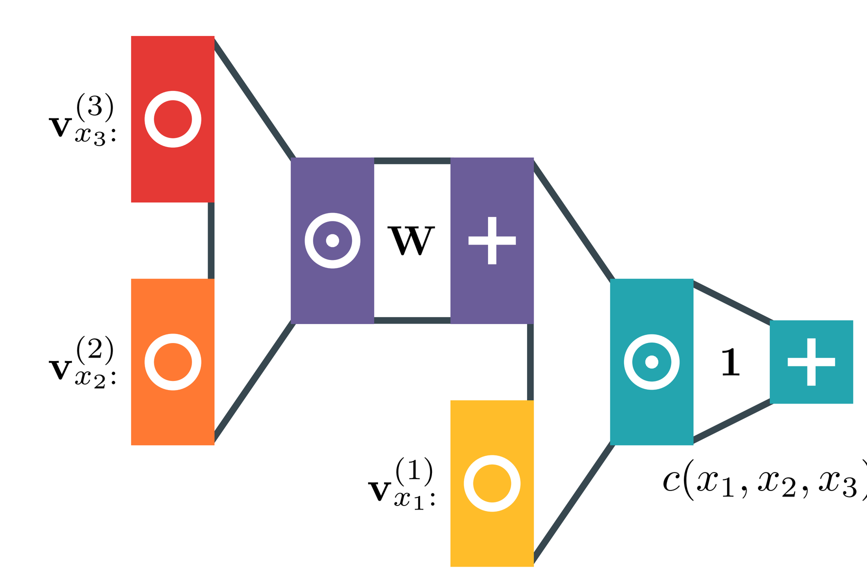
E.g., multilinearity unlocks polytime marginalization [1] [2]

Tucker layer



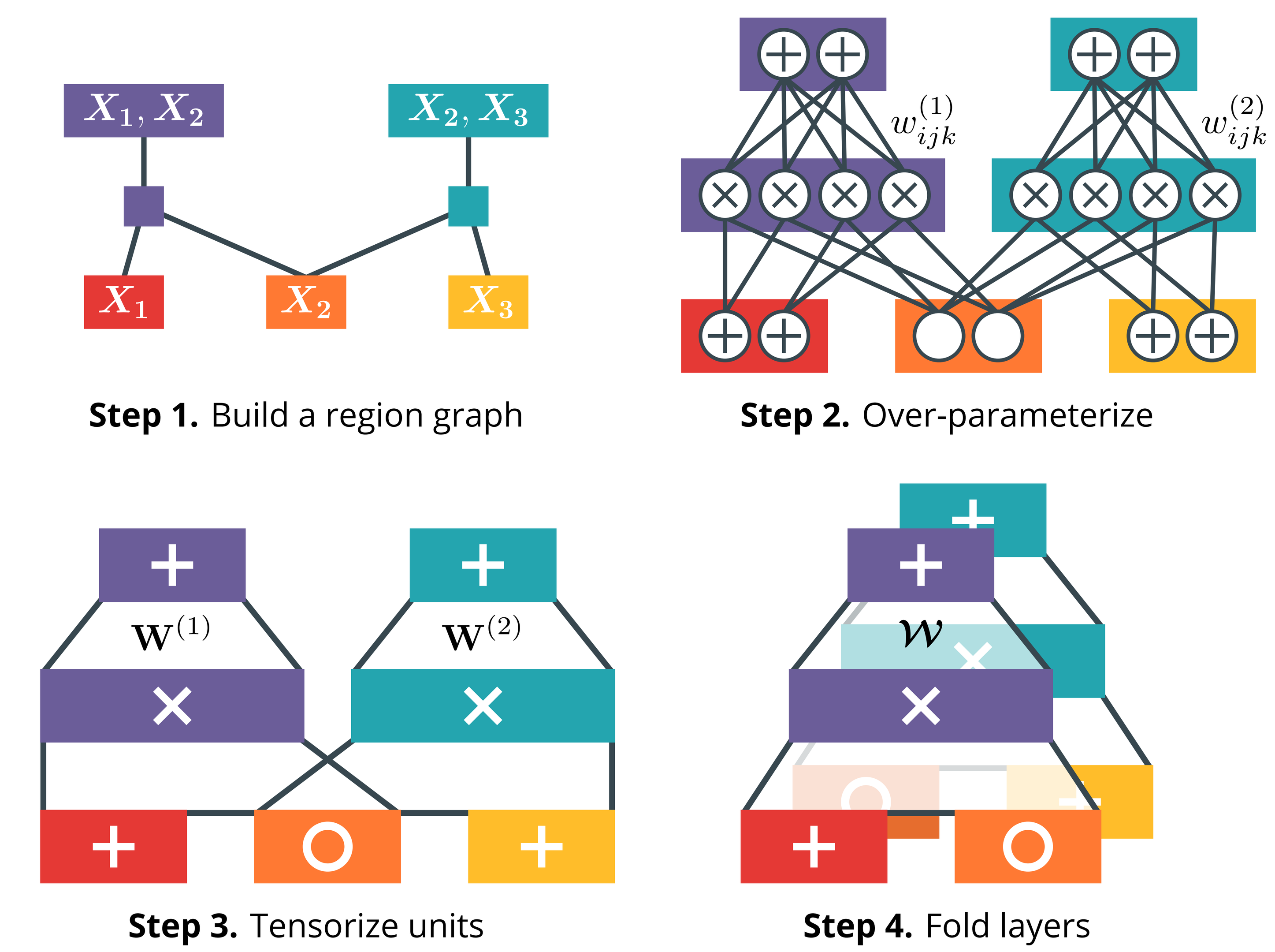
From known factorizations to new hierarchical ones...





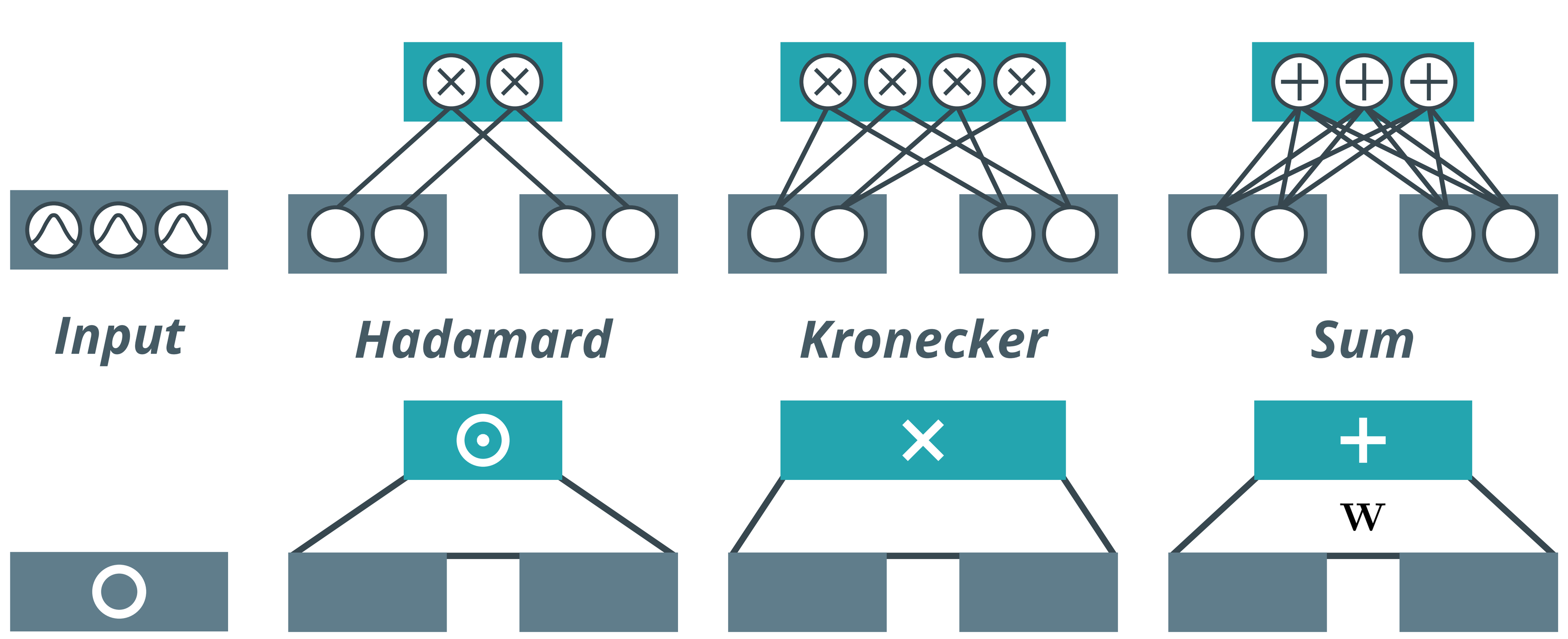
$$t_{x_1x_2x_3} \approx \sum_{r_1=1}^{R} \sum_{r_2=1}^{R} a^{(1)}_{x_1r_1}\, a^{(2)}_{r_1x_2r_2}\, a^{(3)}_{r_2x_3}$$

...strictly generalizing tensor network factorizations [3]



A pipeline for building tensorized circuits in PyTorch

PC Architecture	Region Graph	Sum-Product Layer	Fold
Poon&Domingos (Poon & Domingos, 2011)	PD	$\mathrm{CP}^{\top}$	✗
RAT-SPN (Peharz et al., 2020c)	RND	Tucker	✗
EiNet (Peharz et al., 2020a)	{ RND, PD }	Tucker	✓
HCLT (Liu & Van den Broeck, 2021b)	CL	$\mathrm{CP}^{\top}$	✓
HMM/$\mathrm{MPS}_{\mathbb{R}\geqslant 0}$ (Glasser et al., 2019)	LT	$\mathrm{CP}^{\top}$	✗
BM (Han et al., 2018)	LT	$\mathrm{CP}^{\top}$	✗
TTDE (Novikov et al., 2021)	LT	$\mathrm{CP}^{\top}$	✗
NPC^2 (Loconte et al., 2024)	{ LT, RND }	{ $\mathrm{CP}^{\top}$, Tucker }	✓
TTN (Cheng et al., 2019)	QT-2	Tucker	✗
Mix & Match (our pipeline)	{ RND, PD, LT, CL, QG, QT-2, QT-4 } ×	{ Tucker, CP, $\mathrm{CP}^{\top}$ } ∪ { CP^{S}, $\mathrm{CP}^{\mathrm{XS}}$ | Fold ✓ } ×	{ ✗, ✓ }



A tensorized "Lego block" language for tensor factorizations
enabling us to flexibly build tensor factorizations as neural nets

We present 7+ research opportunities by exchanging theory and practice between the two communities

— *"Derive efficient operators over tensors"* [2]
— *"Use tensor factorizations as circuit layers"* [4]
— *"Impose structural sparsity using constraints"* [5]
— *"Parametrize factorizations via neural networks"* [6]
— ***... and many more in the paper!***

cirkit

We have an OSS lib!

References

[1] Y. Choi, A. Vergari, and G. Van den Broeck. *Probabilistic Circuits: A Unifying Framework for Tractable Probabilistic Modeling*. Tech. rep. University of California, Los Angeles (UCLA), 2020.
[2] A. Vergari et al. "A Compositional Atlas of Tractable Circuit Operations for Probabilistic Inference". In: *Advances in Neural Information Processing Systems 34 (NeurIPS)*. Curran Associates, Inc., 2021, pp. 13189–13201.
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[4] H. Zhang et al. "Scaling up Probabilistic Circuits via Monarch Transformations". In: *1st workshop on Connecting Low-Rank Factorizations in AI (CoLoRAI)*. 2025.
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[6] X. Shao et al. "Conditional sum-product networks: Imposing structure on deep probabilistic architectures". In: *International Conference on Probabilistic Graphical Models*. PMLR. 2020, pp. 401–412.